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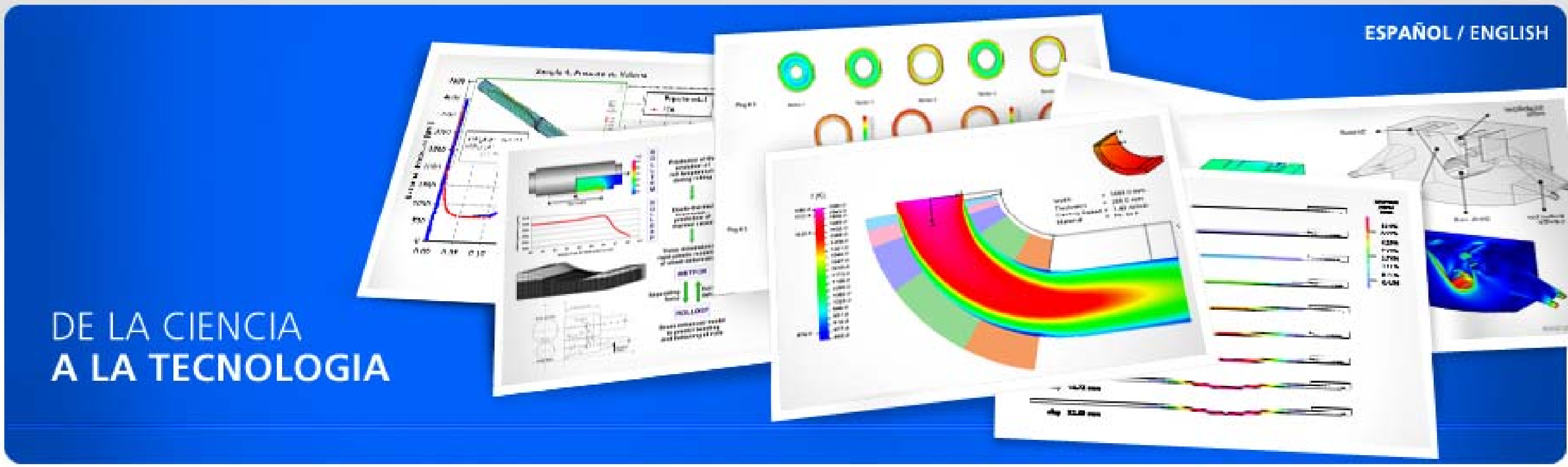

Advanced Topics in Computational Solid Mechanics. Industrial Applications

Section 9 : Tracking Nonlinear Equilibrium paths: The Riks Method

Eduardo N. Dvorkin

Stanford University
Mechanical Engineering
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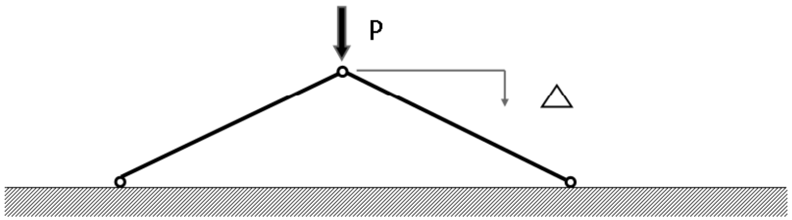
Load Control Solution

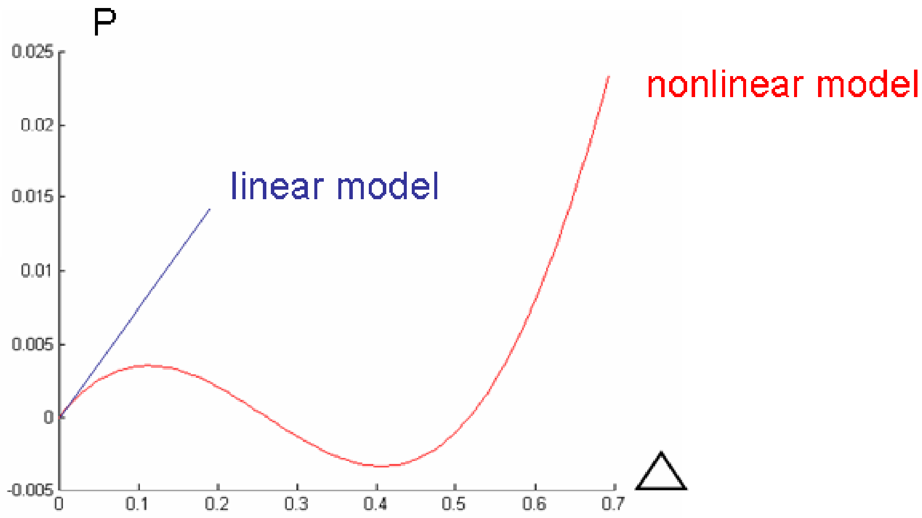
$$^{t+\Delta t}\mathbf{R} - {}^{t+\Delta t}\mathbf{F} = \mathbf{0}$$

$$^{t+\Delta t}\mathbf{K}^{(i-1)}\Delta\mathbf{U}^{(i)} = {}^{t+\Delta t}\mathbf{R} - {}^{t+\Delta t}\mathbf{F}^{(i-1)}$$

$$^{t+\Delta t}\mathbf{U}^{(i)} = {}^{t+\Delta t}\mathbf{U}^{(i-1)} + \Delta\mathbf{U}^{(i)}$$

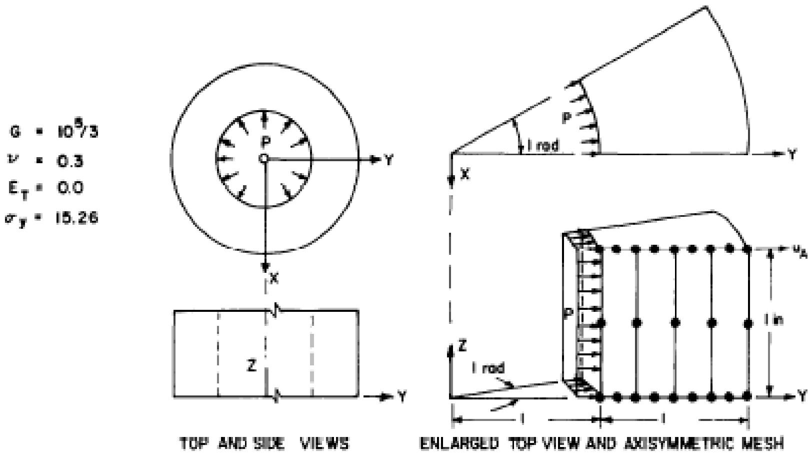
Fails in:

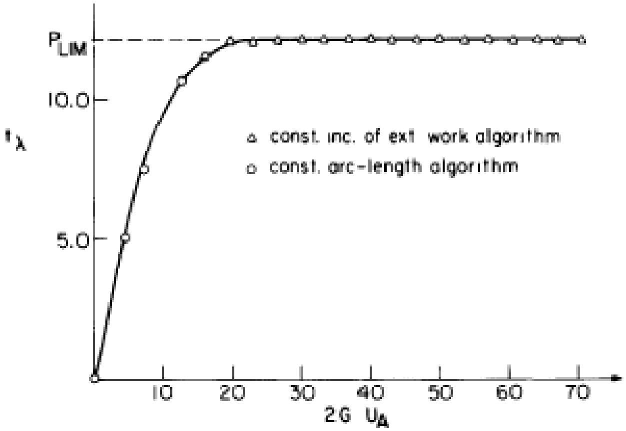


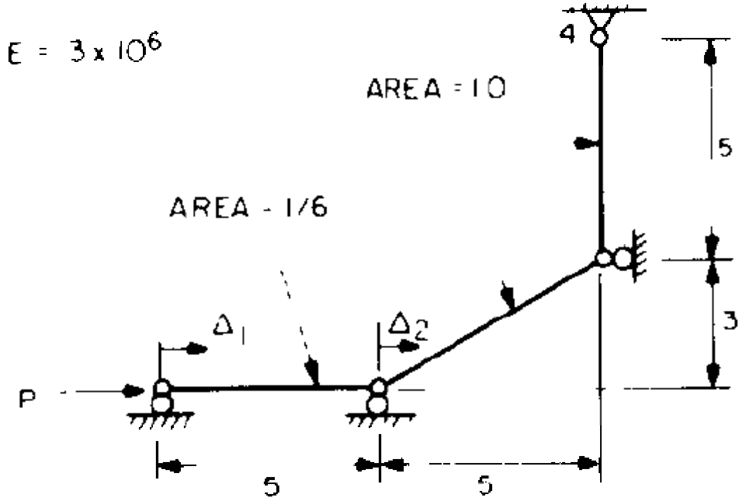


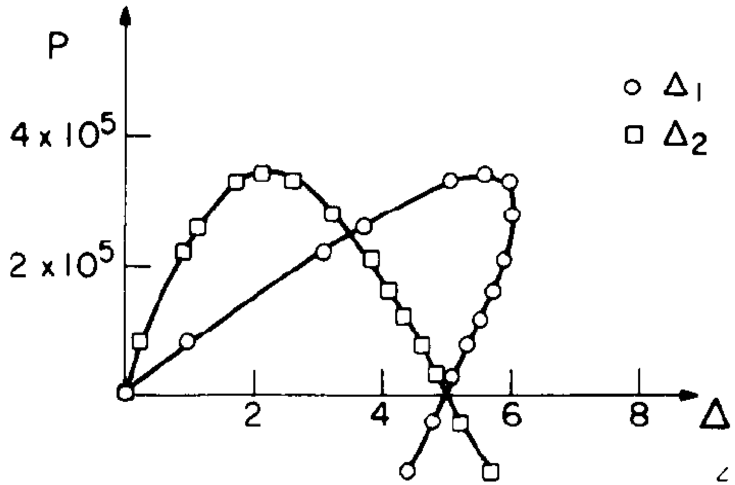
Load Control Failures

Elasto-perfectly plastic material.
Plane Strain conditions. Von-mises yield condition



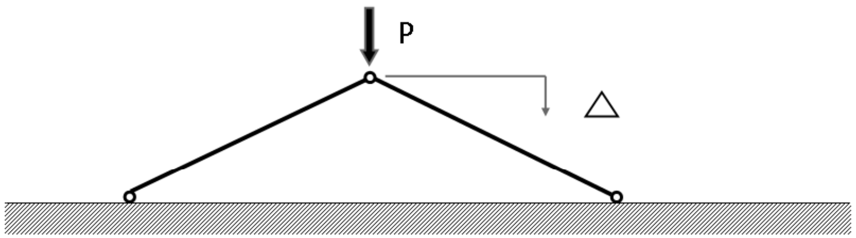


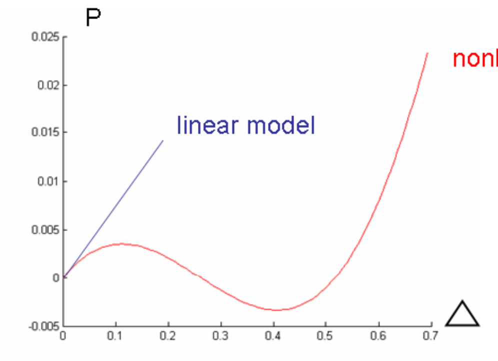


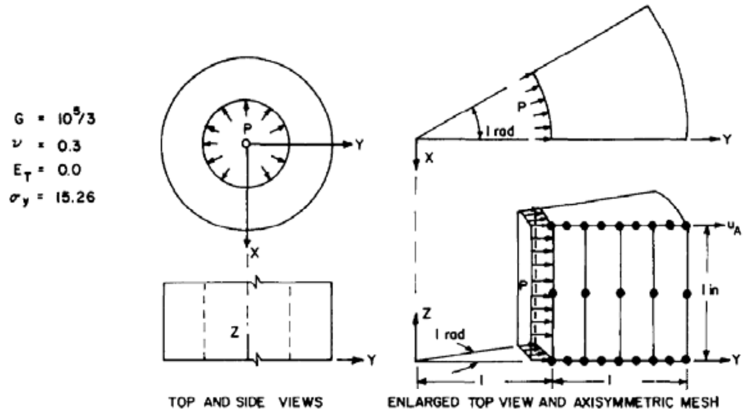


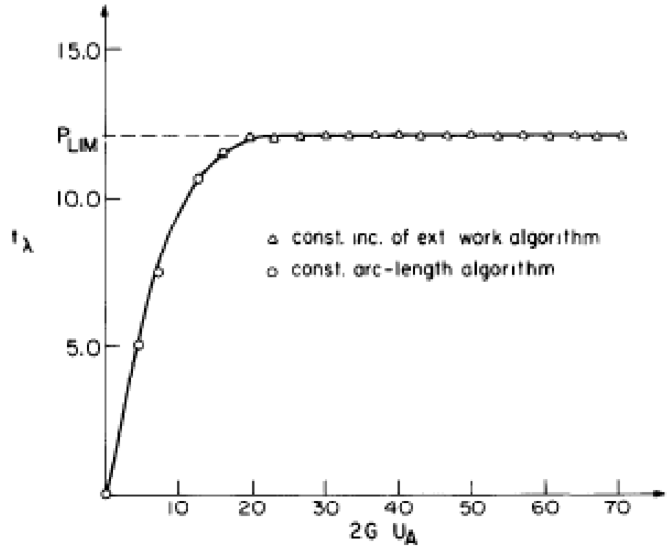
Displacement Control Solution

o.k. for:

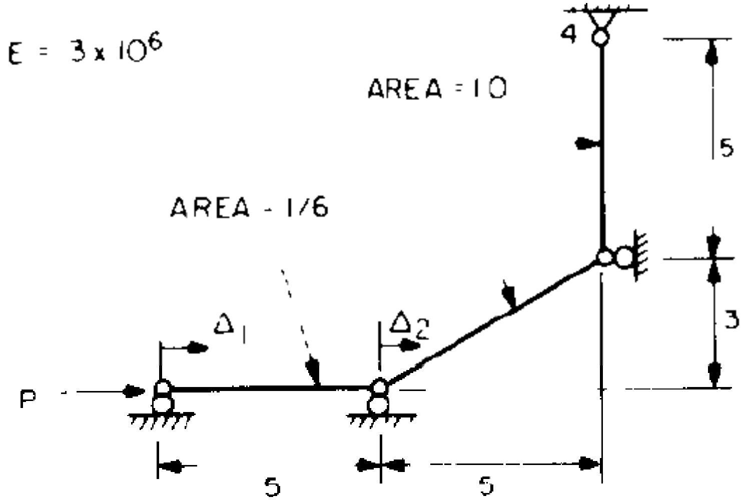




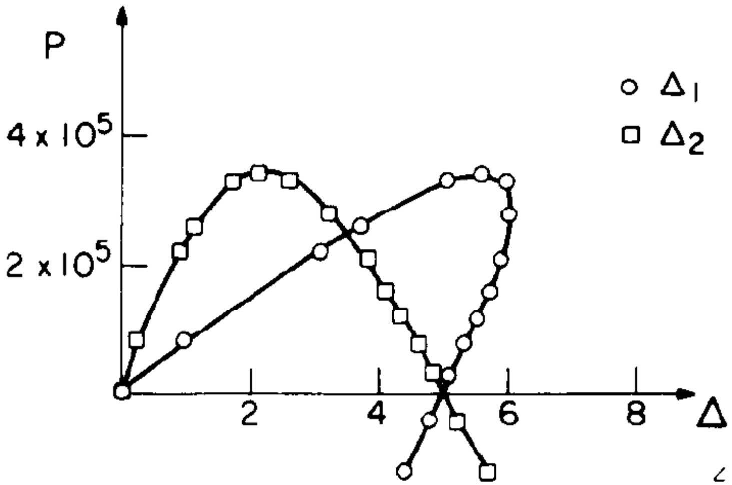




Displacement Control Solution



Fails for:



The Riks Method:
Iterate in the Load – Displacement Space

$$ {}^{\tau}\mathbf{K}\,\Delta\mathbf{U}^{(i)} = {}^{t+\Delta t}\lambda\,\mathbf{R} - {}^{t+\Delta t}\mathbf{F}^{(i-1)} $$

$$ {}^{\tau}\mathbf{K}\,\Delta\mathbf{U}^{(i)} = ({}^{t+\Delta t}\lambda^{(i-1)} + \Delta\lambda^{(i)})\mathbf{R} - {}^{t+\Delta t}\mathbf{F}^{(i-1)} $$

and an additional equation is employed to constrain the length of the load step

$$ f(\Delta\lambda^{(i)}, \Delta\mathbf{U}^{(i)}) = 0 $$

The Riks Method: Arc Length

We use the spherical constant arc-length in the response regions far from the critical points

$$\{({}^{t+\Delta t}\lambda^{(i-1)} - {}^{t}\lambda) + \Delta\lambda^{(i)}\}^2 + \mathbf{U}^{(i)T}\,\mathbf{U}^{(i)} = \Delta l^2 \qquad (7)$$

$$\mathbf{U}^{(i)} = {}^{t+\Delta t}\mathbf{U}^{(i)} - {}^{t}\mathbf{U}$$

where Δl is the arc-length.

The Riks Method: Constant Increment of External Work

The scheme of constant increment of external work is used near the critical points. In this case eqn (6) is for the first iteration,

$$({}^{t}\lambda + \tfrac{1}{2}\,\Delta\lambda^{(1)})\mathbf{R}^{T}\,\Delta\mathbf{U}^{(1)} = W \qquad (8a)$$

and for the next iterations,

$$({}^{t+\Delta t}\lambda^{(i-1)} + \tfrac{1}{2}\,\Delta\lambda^{(i)})\mathbf{R}^{T}\,\Delta\mathbf{U}^{(i)} = 0 \qquad i = 2, 3, \ldots \qquad (8b)$$

The Riks Method:Practical Implementation

(1) The user inputs the reference load distribution, which corresponds to the vector R. This load distribution is varied proportionally during the analysis and can be due to distributed and concentrated loads.

(2) The user specifies the displacement at a node corresponding to the first load level (i.e. corresponding to ${}^{\Delta t}\lambda$). We denote this displacement as ${}^{\Delta t}U_k{}^*$. Here it is deemed that to start the incremental solution it is easier to specify the displacement at a node, that the user selects, than the intensity (${}^{\Delta t}\lambda$) of the loads.

(3) The displacements corresponding to time Δt determined by the specified displacement ${}^{\Delta t}U_k{}^*$ also limit the size of any subsequent load change per step, because the user specifies a constant α and the algorithm assures that

$$\|\mathbf{U}\| \leq \alpha \, \|{}^{\Delta t}\mathbf{U}\| \dagger \qquad (9)$$

where $\mathbf{U}$ is the displacement increment in any load step and ${}^{\Delta t}\mathbf{U}$ are the displacements corresponding to time Δt.

The Riks Method:Practical Implementation

First step

$$^{0}\mathbf{K}\ \Delta\mathbf{U}^{(i)} = ({}^{\Delta t}\lambda^{(i-1)} + \Delta\lambda^{(i)})\mathbf{R} - {}^{\Delta t}\mathbf{F}^{(i-1)}$$

$$^{\Delta t}\lambda^{(0)} = 0;\ {}^{\Delta t}\mathbf{F}^{(0)} = \mathbf{0}$$

$$^{0}\mathbf{K}\ \Delta\mathbf{U}^{(1)} = \mathbf{R}$$

$$^{\Delta t}\lambda^{(1)} = \frac{^{\Delta t}U_k^{*}}{\Delta U_k^{(1)}};\ {}^{\Delta t}\mathbf{U}^{(1)} = {}^{\Delta t}\lambda^{(1)}\ \Delta\mathbf{U}^{(1)}$$

i=2,3,….

$$^{0}\mathbf{K}\ \Delta\bar{\mathbf{U}}^{(i)} = {}^{\Delta t}\lambda^{(i-1)}\mathbf{R} - {}^{\Delta t}\mathbf{F}^{(i-1)}$$

$$^{0}\mathbf{K}\ \Delta\bar{\bar{\mathbf{U}}}^{(i)} = \Delta\lambda^{(i)}\mathbf{R}$$

$$\Delta\bar{\bar{\mathbf{U}}}^{(i)} = \Delta\lambda^{(i)}\ \Delta\mathbf{U}^{(1)}.$$

$$\Delta\lambda^{(i)} = -\frac{\Delta\bar{U}_k^{(i)}}{\Delta U_k^{(1)}}$$

$$^{\Delta t}\lambda^{(i)} = {}^{\Delta t}\lambda^{(i-1)} + \Delta\lambda^{(i)}$$

$$^{\Delta t}\mathbf{U}^{(i)} = {}^{\Delta t}\mathbf{U}^{(i-1)} + \Delta\mathbf{U}^{(i)}$$

$$\Delta\mathbf{U}^{(i)} = \Delta\bar{\mathbf{U}}^{(i)} + \Delta\lambda^{(i)}\ \Delta\mathbf{U}^{(1)}.$$

The Riks Method:Practical Implementation

2Δt; 3Δt; … (arc length)

$$\Delta l = \beta \sqrt{\mathbf{U}^T\mathbf{U} + \lambda^2}$$

$$\mathbf{U} = {}^t\mathbf{U} - {}^{t-\Delta t}\mathbf{U};\ \lambda = {}^t\lambda - {}^{t-\Delta t}\lambda$$

$${}^t\mathbf{K}\ \Delta\bar{\mathbf{U}}^{(1)} = {}^t\lambda\ \mathbf{R} - {}^t\mathbf{F}$$

$$\Delta\mathbf{U}^{(1)} = \Delta\bar{\mathbf{U}}^{(1)} + \Delta\lambda^{(1)}\ \Delta\bar{\bar{\mathbf{U}}}^{(1)}$$

$${}^t\mathbf{K}\ \Delta\bar{\bar{\mathbf{U}}}^{(1)} = \mathbf{R}$$

$${}^{t+\Delta t}\mathbf{U}^{(1)} = {}^t\mathbf{U} + \Delta\mathbf{U}^{(1)};\ {}^{t+\Delta t}\lambda^{(1)} = {}^t\lambda + \Delta\lambda^{(1)}$$

$$\Delta\mathbf{U}^{(1)T}\ \Delta\mathbf{U}^{(1)} + (\Delta\lambda^{(1)})^2 = \Delta l^2.$$

select one of the roots for Δλ

i=2,3,….

$${}^\tau\mathbf{K}\ \Delta\bar{\mathbf{U}}^{(i)} = {}^{t+\Delta t}\lambda^{(i-1)}\mathbf{R} - {}^{t+\Delta t}\mathbf{F}^{(i-1)}$$

$${}^{t+\Delta t}\mathbf{U}^{(i)} = {}^{t+\Delta t}\mathbf{U}^{(i-1)} + \Delta\bar{\mathbf{U}}^{(i)} + \Delta\lambda^{(i)}\ \Delta\bar{\bar{\mathbf{U}}}^{(1)} \qquad {}^{t+\Delta t}\lambda^{(i)} = {}^{t+\Delta t}\lambda^{(i-1)} + \Delta\lambda^{(i)}$$

The Riks Method:Practical Implementation

$$\Delta l\big|_{\text{new}} = \Delta l\big|_{\text{old}} \sqrt{\frac{N_1}{N_2}} \frac{\|\mathbf{U}\|_{\text{allowable}}}{\|\mathbf{U}\|_{\text{actual}}}$$

The Riks Method:Practical Implementation

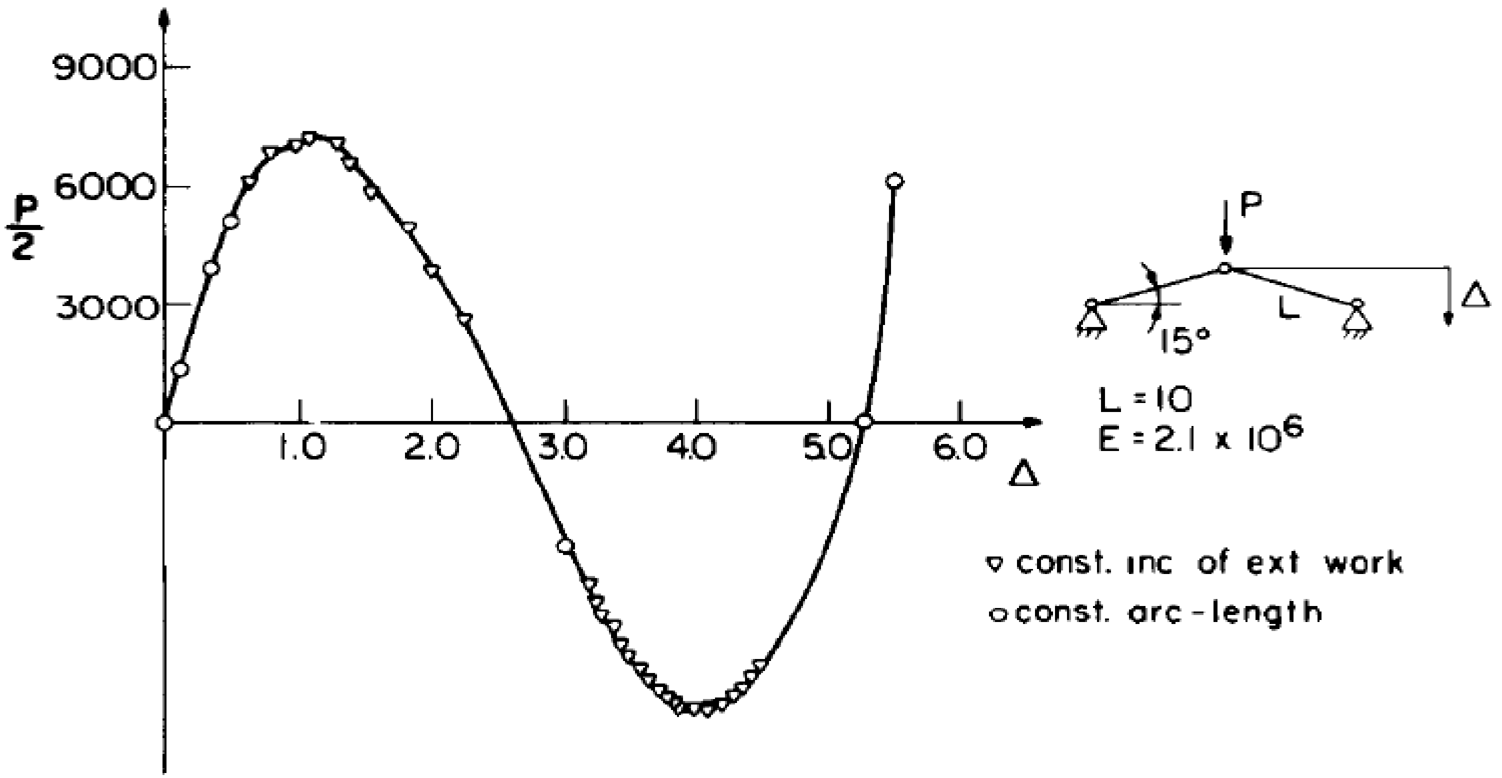
2Δt; 3Δt; … (constant increment of external work)

$$^{t+\Delta t}W = \beta^{t}\ {}^{t}W$$

$$({}^{t}\lambda + \tfrac{1}{2}\Delta\lambda^{(1)})\mathbf{R}^{T}\Delta\mathbf{U}^{(1)} = W$$

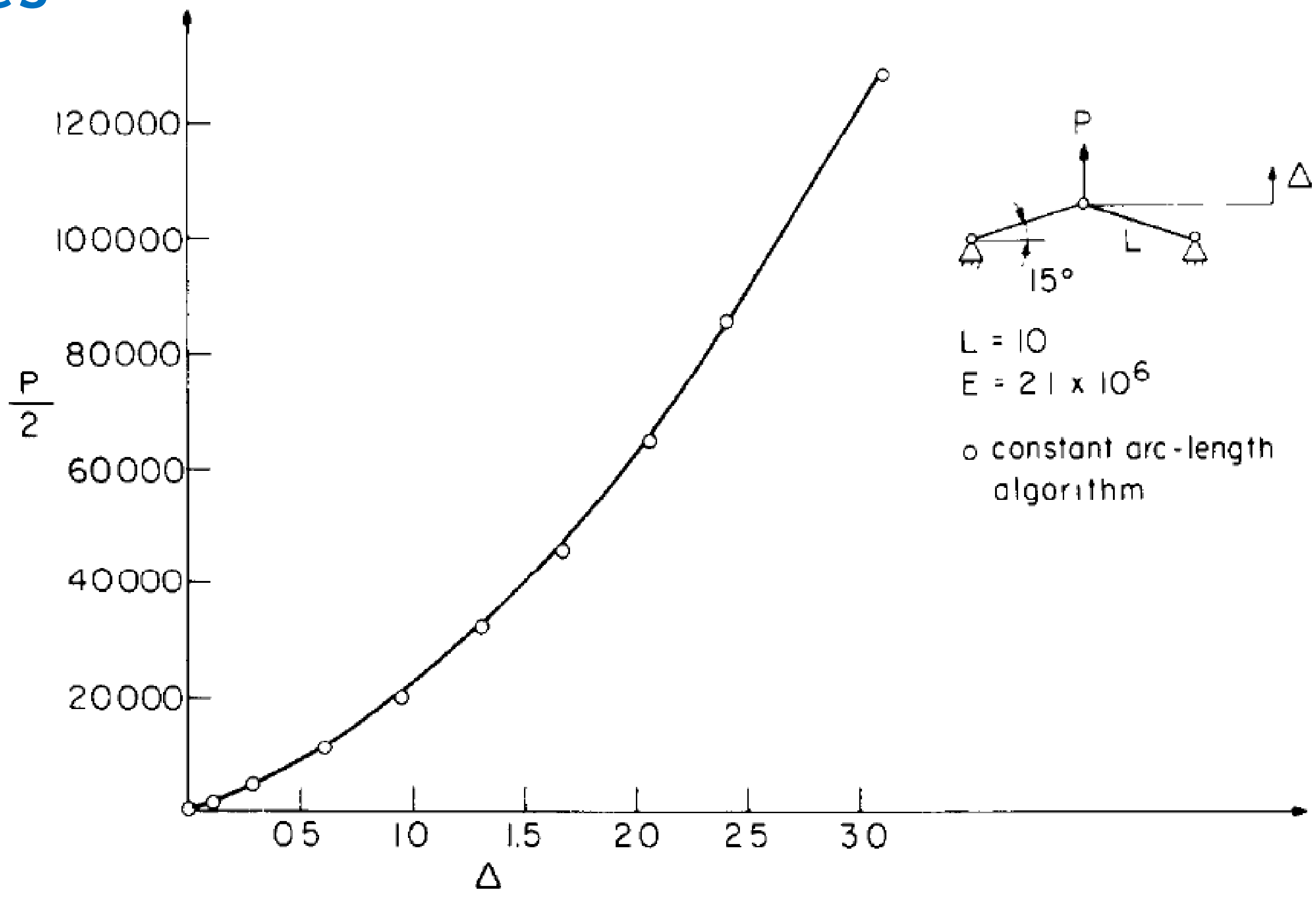
$$({}^{t+\Delta t}\lambda^{(i-1)} + \tfrac{1}{2}\Delta\lambda^{(i)})\mathbf{R}^{T}\Delta\mathbf{U}^{(i)} = 0$$

Examples



9000
6000
3000
P/2
1.0
2.0
3.0
4.0
5.0
6.0
Δ
P
L
Δ
15°
L = 10
E = 2.1 x 10^6
▽ const. inc of ext work
○ const. arc-length

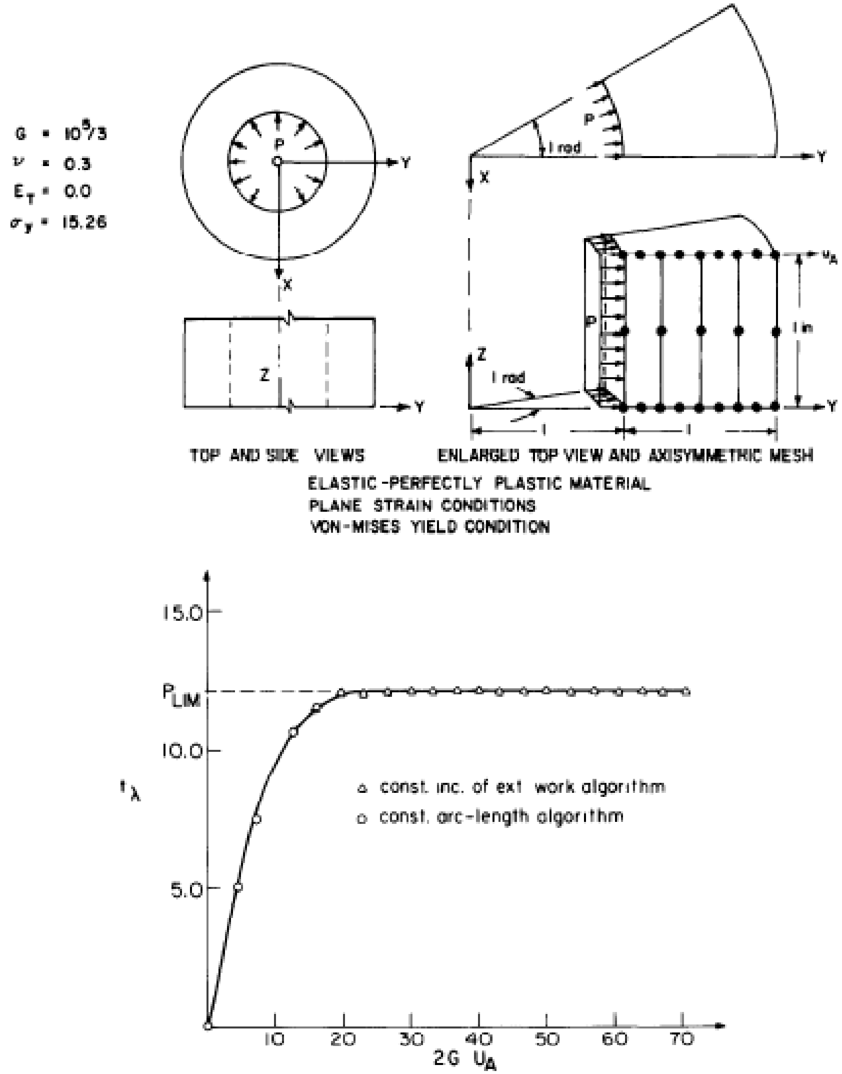
Examples



b) Stiffening problem

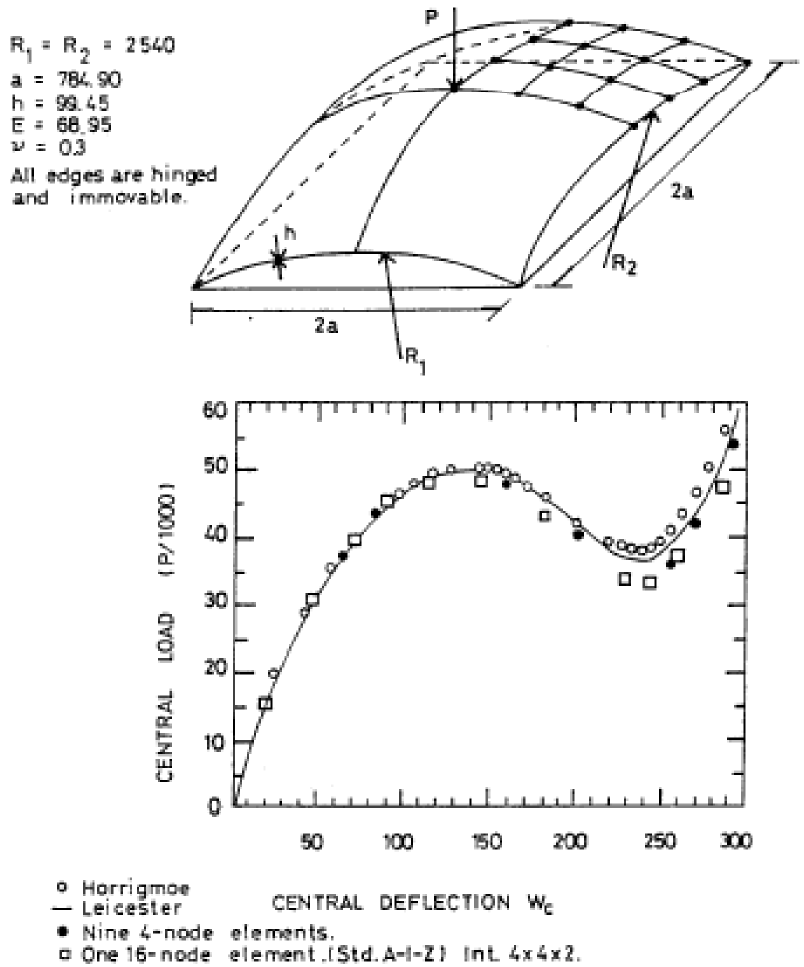
Examples

Thick-walled elastic perfectly plastic cylinder



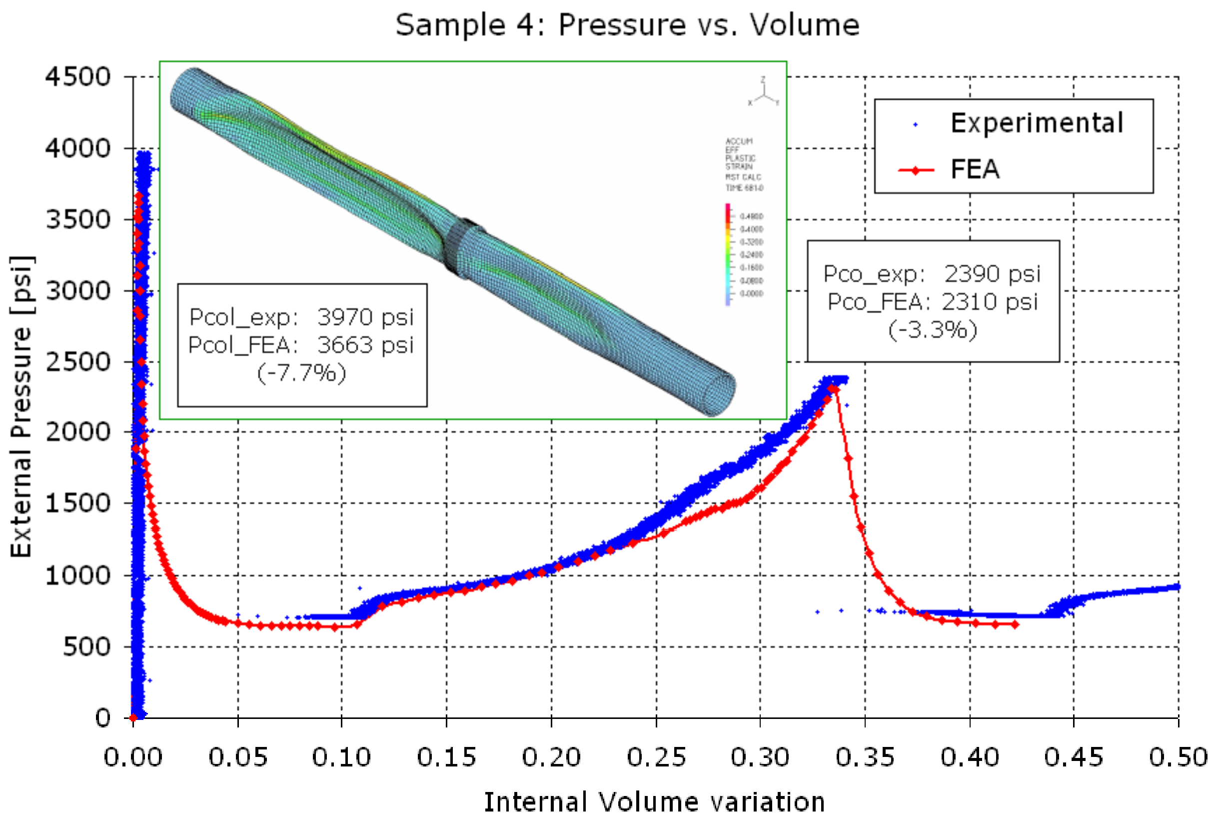
Examples

Nonlinear spherical shell (MITC4)



Examples

Collapse



Iterative Methods

We have plenty of freedom to select the matrix

because we just have to get,

$$\left[{}^{t}R\right] - \left[{}^{t}F\right]_{(k-1)} \quad < \quad RTOL$$

independently of the iteration path.

Iterative methods: full Newton, modified Newton, BFGS.
Combine with line searches.

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Iteration Tolerances

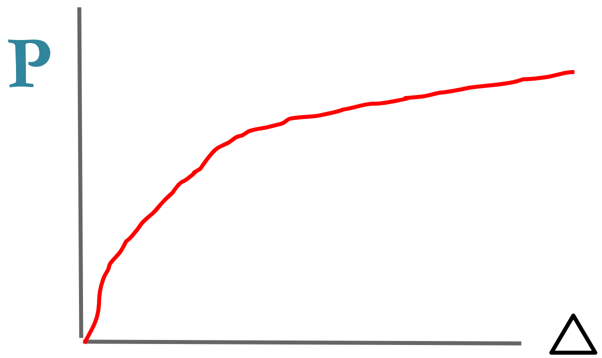
A fundamental decision:

- If too restrictive we may not get a solution
- If too ample we may get a very bad solution

$$
\begin{aligned}
\left[{}^{t}R\right] - \left[{}^{t}F\right]_{(k-1)} &< RTOL \\
\left\| \left[\Delta \underline{U}\right]_{(k)} \right\| &< UTOL \\
\left(\left[\Delta \underline{U}\right]_{(k)}\right)^{T} \left(\left[{}^{t}R\right] - \left[{}^{t}F\right]_{(k-1)}\right) &< ETOL
\end{aligned}
$$

Iteration Tolerances

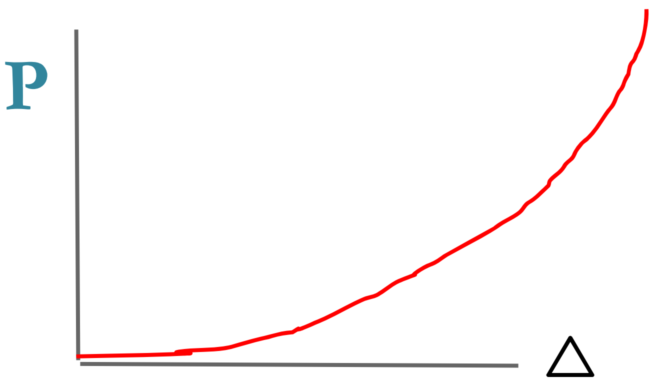
Examples:



<u>Softening problem</u>

$$\left[{}^{t}R\right] - \left[{}^{t}F\right]_{(k-1)} < RTOL$$

Alone is not enough



<u>Stiffening problem</u>

$$\left\| \left[\Delta \underline{U}\right]_{(k)} \right\| < UTOL$$

Alone is not enough